Study Guide for Midterm 2

Topics: 3.1–3.8, 4.1–4.3, 6.1

§ 3.1 – Homogeneous vs. Nonhomogeneous linear 2nd order equations; Characteristic equations; Solution with distinct real roots for $ay'' + by' + cy = 0$.

Problems for Review: p. 144 11–15, 24, 27

§ 3.2 – Wronskian, fundamental set of solutions, ~~EXIST~~ Existence and Uniqueness Theorem.

Problems for Review: p. 158 ~~8~~, 8, 10, 15, 26

§ 3.3 – Solving $ay'' + by + cy = 0$ where characteristic equation $ar^2 + br + c = 0$ has complex roots.

Problems for Review: p. 164 13–16, 20, 21, 27

* p. 166 Euler equations – use $y = t^r$ as model solution to find r as root of characteristic equation.

Problems for Review: 35–38

Good for understanding but not directly testable.

§3.4 – Solve $ay'' + by' + cy = 0$ in case characteristic equation has a repeated roots, i.e., $ar^2 + br + c = a(r+\gamma)^2$

Problems for Review: p 173 7, 9, 10, 11, 13, 16

* Reduction of Order –

Don't forget about this!

$y_1(t)$ is a solution, find y_2 by guessing $y_2 = v \cdot y_1$ and solving a lower order equation in terms of $w = v'$.

Problems for Review: p. 174 24, 26, 27, 28

§3.5 – Nonhomogeneous equations, Method of Undetermined Coefficients aka "Guess and Check".

Refer to Box on p.182 for correct guessing.

Best for: $g(t)$ is a polynomial, $g(t)$ is $\sin(t)$ or $\cos(t)$

Problems for Review: p. 184 9, 12, 23, 24

§3.6 – Nonhomogenous equations; Variation of Parameters

Box on p. 190

$$Y(t) = -y_1(t)\int_{t_0}^{t} \frac{y_2(s)\,g(s)}{W(s)}\,ds + y_2(t)\int_{t_0}^{t} \frac{y_1(s)\,g(s)}{W(s)}\,ds$$

$W(s) = W(y_1, y_2)(s)$ as a function of s.

$$= \begin{vmatrix} y_1(s) & y_1'(s) \\ y_2(s) & y_2'(s) \end{vmatrix}$$

Problems for Review: p. 190 7, 11, 14, 17

§3.7 + 3.8 – Springs

Spring equation – $mu'' + \gamma u' + ku = F(t)$

Problems for Review: p. 204 7, 10, 11, p. 217 11, 12

§ 4.1, 4.2 Higher Order Linear Equations.

characteristic equation, Existence and Uniqueness, Finding solutions from roots of characteristic equations, * multiplicity of roots.

Problems for Review: p. 234 17, 21, 30, 32.

§ 4.3 Method of Undetermined Coefficients.

- Pretty much the same as §3.5

Problems for Review: p. 239 11, 13, 14, 15

§ 6.1 – Just know how to compute a Laplace transform.

$$\mathcal{L}(f)(s) = \int_0^\infty f(t)\, e^{-st}\, dt.$$

Tasks you are expected to be able to complete.

A. Solve any $ay'' + by' + cy = 0$ <u>Cold</u>.

Characteristic equation: $ar^2 + br + c = 0$

* $a(r - r_1)(r - r_2)$ $\quad r_1 \neq r_2$ real, $b^2 - 4ac > 0$

$$y_1 = e^{r_1 t}, \quad y_2 = e^{r_2 t}$$

* $a(r - r_1)(r - r_2)$ $\quad r_1, r_2$ complex, $b^2 - 4ac < 0$

$$r_1 = \alpha + i\beta, \quad r_2 = \alpha - i\beta$$

$$y_1 = e^{\alpha t}\cos(\beta t) \qquad y_2 = e^{\alpha t}\sin(\beta t)$$

* $a(r - r_1)^2$ $\quad r_1$ real, $b^2 - 4ac = 0$

$$y_1 = e^{r_1 t}, \quad y_2 = te^{r_1 t}$$

B. Solve any higher order homogeneous equation using roots of characteristic equation.

C. Reduction of Order

$$y'' + p(t)y' + q(t)y = 0$$

y_1 is a solution.

Set $y_2 = v \cdot y_1$

$$y_2' = v'y_1 + vy_1'$$

$$y_2'' = v''y_1 + 2v'y_1' + vy_1''$$

Plug y_2 into equation to get something of the form

Done correctly there is no "v" term

$$P(t)v'' + Q(t)v' = 0$$

set $w = v'$ and solve 1st order equation

$$P(t)w' + Q(t)w = 0$$

then integrate w to get v.

D. Solve a lame story problem involving a spring

Really, I am going to literally copy a spring problem from the book only changing numbers.

E. Solve a Nonhomogenous Equation.

You are free to use either method.

However, one method may be clearly preferable and knowing that and demonstrating you know that counts.

F. Solve a higher order linear equation using the characteristic equations.

May be homogenous or nonhomogenous.

G. Compute a Laplace Transform.